Estimation of Weapon-Radius vs Maneuverability Tradeoff for Air-to-Air Combat

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A chase in a horizontal plane between a pursuer with a large capture radius and a more maneuverable evading vehicle is examined with constant-speed vehicle models. An approximation to the "sidestepping" maneuver of the Homicidal Chauffeur Game is modified to account for the effect of evader turning rate, and an estimate of capture radius required is so obtained which agrees remarkably well with Cockayne's point-capture result. The maneuver assumes central importance for barrier surfaces appearing in the Game of Two Cars. Results are given for required weapon capture-radius in terms of the maneuverability of the two vehicles. Some calculations of capture radius are presented.

Introduction

A CHASE in a horizontal plane using the Game-of-Two-Cars model is examined in the following. An approximation to the "sidestepping" maneuver of the Homicidal Chauffeur Game[1-3] is obtained and modified to account for the effect of evader turning rate, and an estimate of capture radius required presented. This turns out to agree remarkably well with Cockayne's point-capture result.[4] It develops that the capture-radius required for the breakaway maneuver approximating the sidestepping, as it relates to the actual capture radius, assumes central importance for barrier surfaces appearing in the Game of Two Cars.

A procedure for calculation of capture radius required to overcome a maneuverability deficiency will be presented along with some example computations using data for two aircraft,[5,6] one a version of the F-4, the other a hypothetical Mach 3 aircraft, each taking in turn the roles of pursuer and evader.

Analysis of Terminal Maneuver

Vehicle 2, the pursuer, is assumed to possess at least a slight speed advantage over evading Vehicle 1. Vehicle 2 has a limit ω_2 to its heading angular rate, whereas Vehicle 1 can change heading instantaneously. These are the vehicle and pedestrian models of Isaacs' Homicidal Chauffeur Game.[1-3] For large capture radius, the intricacies of maneuvering disappear, and attention focuses on a terminal "sidestepping" maneuver. Initially the pursuer and evader are arrayed line-astern. The evader allows the pursuer to approach to a distance $a = V_1/\omega_2$, then "sidesteps." An evasive maneuver any earlier would be unproductive, as pursuit navigation would be possible within the pursuer's angular rate limit.

In the approximation of present interest, the evasive maneuver is a 90° turn and the pursuit path a parabola as shown in Fig. 1. Refinements on this modeling are possible, but of questionable worth considering the shakiness of the basic constant-speed assumption for air-combat applications. The assumption that the pursuer's heading change is small, $\omega_2 t \ll 1$, so that $\cos \omega_2 t \cong 1$, and $\sin \omega_2 t \cong \omega_2 t$, is incorporated in the following equations

$$x_1 = a + V_1 t \ \cos\chi_1 \tag{1}$$

$$y_1 = V_1 t \ \sin\chi_1 \tag{2}$$

$$x_2 = V_2 t \tag{3}$$

$$y_2 = (V_2 \omega_2 t^2/2) \tag{4}$$

With $\chi_1 = 90°$, the magnitude of the relative distance between vehicles is

$$R = \left\{ \left[V_1 t - \frac{V_2 \omega_2 t^2}{2} \right]^2 + [a - V_2 t]^2 \right\}^{1/2} \tag{5}$$

At time, a/V_2, this furnishes an approximation to the closest approach distance,

$$R_c = \left(V_1 - \frac{\omega_2 a}{2} \right) \frac{a}{V_2} = \frac{V_1^2}{2\omega_2 V_2} \tag{6}$$

(where $a = V_1/\omega_2$ has been used).

A correction for the evader's time-to-turn 90° can be introduced by using an average value for $\sin \chi_1$ during this interval $\Delta = (\pi/2\omega_1)$, yielding

$$\bar{R}_c = \frac{V_1^2}{2V_2\omega_2} - \frac{\pi}{2}\left(1 - \frac{\sqrt{2}}{2}\right)\frac{V_1}{\omega_1} \tag{7}$$

One might expect good results from such a correction for evader's turn rate high relative to the pursuer's; however, it turns out that Eq. (7) furnishes a good prediction also for a case in which the turn rates are of the same order. This is the special case of point capture (zero radius) studied by Cockayne, the only case for which an "if-and-only-if" capture criterion for the Game of Two Cars is presently known.[4] Cockayne's result is that capture eventually takes place with optimal play if and only if

$$V_2 > V_1 \tag{8}$$

and

$$V_2\omega_2 \geq V_1\omega_1 \tag{9}$$

These criteria correspond to a margin of superiority in speed, and equality or superiority in normal acceleration. They imply that capture radius required should fall to zero as pursuer's acceleration $V_2\omega_2$ approaches the evader's, $V_1\omega_1$, from below. It is noted that Eq. (7) very nearly duplicates this behavior. It should be noted (and has been, by a reviewer) that the strange and wonderful mixture of approximations employed has the pursuer leading the evader during the early part of the breakaway; but no matter.

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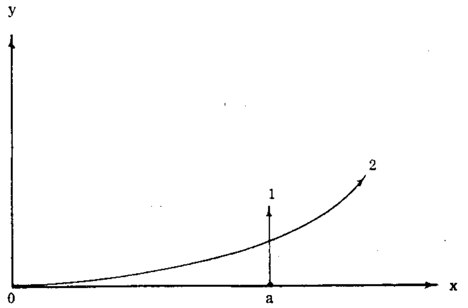

Fig. 1 Sketch.

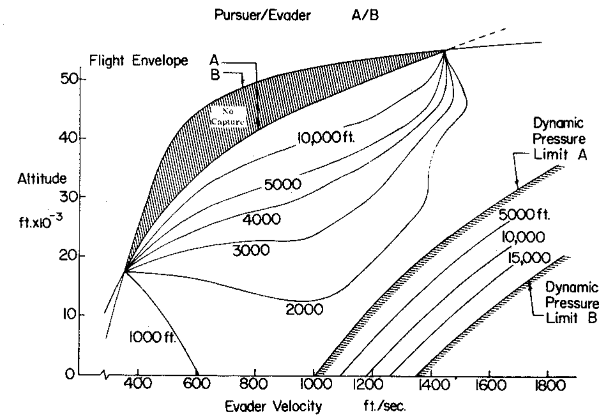

Fig. 2 Capture radius required to counter evasive break – equal airspeeds and altitudes – pursuer/evader A/B.

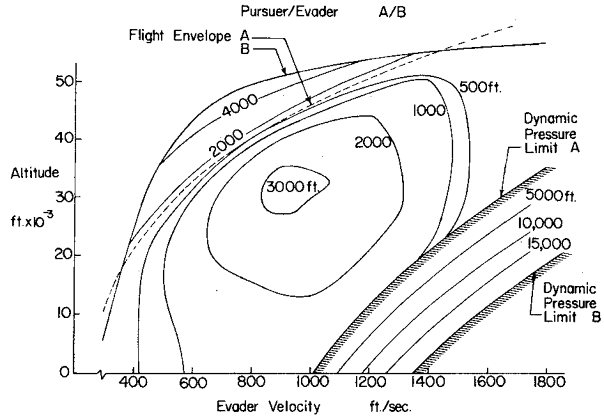

Fig. 3 Capture radius required to counter evasive break – equal energies – noncoaltitude pursuit – pursuer/evader A/B.

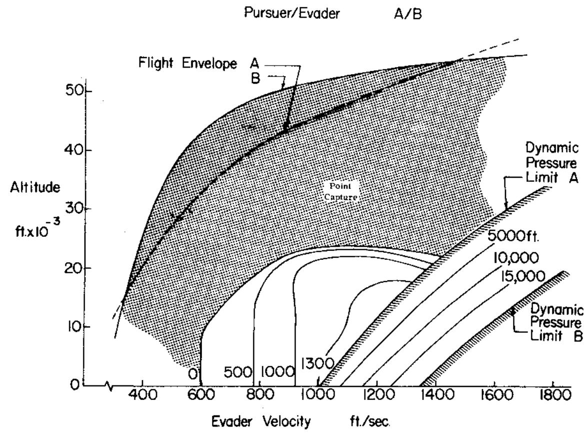

Fig. 4 Capture radius required to counter evasive break – pursuer envelope airspeed – pursuer/evader A/B.

Capture Radius and the Game of Two Cars

That the preceding results for the seemingly rather special rectilinear-chase/breakaway situation are more generally applicable can be argued as follows. Consider a version of the Game of Two Cars in which the index minimaxed is closest-approach distance, time open. Consider a case of interest in which the Cockayne criteria are not met, viz.: the pursuer has a speed advantage, but the evader has a normal acceleration advantage, so that point-capture does not occur except for special initial conditions. A reference maneuver sequence is that of the preceding section, which for well-separated initial conditions consists of a tail-chase followed by a breakaway maneuver by the evader, attempting to emulate the sidestepping pedestrian. For every separation distance, this maneuver sequence yields reference off-boresight and target-aspect angles, and a characteristic miss estimated by Eq. (7). For a general state, the pursuer's situation may be characterized as "better," "worse," or "equivalent" to the reference, depending on comparison of angles. "Worse" could be quantified provisionally as producing a greater miss than the reference, with the pursuer turning toward a collision course, and with optimal evasion. If the pursuer's situation is "worse" in this sense, the pursuer can elect to turn away from the evader, separate well, turn back for another pass; this sequence attains the reference miss, belying the label "worse." Thus the reference miss represents a maximum, given optimal play, specified speeds, and turn rates, and hence is of particular interest for design purposes, i.e., it is, from the pursuer's viewpoint, a worst case.

This has implications for the more conventional minimax-time Game-of-Two-Cars version. If the specified capture radius equals or exceeds the reference miss, eventual capture is assured, and there is no closed barrier surface; if not, capture is assured only for a subset of close-in initial conditions corresponding to angular geometry more favorable to the pursuer than the reference geometry at the same separation distance. For initial points outside the barrier surface which is the boundary of the subset, indefinitely prolonged evasion is possible.

Air-Combat Weapon-Radius Requirements

An approximation scheme that crams real airplane data into the mold of constant-velocity models is of necessity a sweeping one. An attempt is described as follows.

The evader, in anticipation of a pass from behind, prepares for an evasive maneuver consisting of a simple but well-timed break, left or right, at the highest *sustainable* turn rate for the speed and altitude. The evader may or may not have sufficient time to adjust his speed and altitude to a favorable combination before his breakaway, and both cases are of interest; however, in both, it is assumed that the evasive maneuver takes place in the horizontal plane. The pursuer is forced to match the evader's altitude, more or less, any mismatch appearing as an additional component of miss. The pursuer's speed must be at least the lower bound for closure (viz. the evader's speed) which is one extreme of interest, while the other is that speed between the lower bound and the prevailing Mach or dynamic-pressure limit which furnishes the pursuer with maximum sustainable acceleration $V_2\omega_2$. The case of marginal overtaking speed is probably more typical of the rough and tumble of actual air combat than is the well-planned and executed pass for which energy has been stored up.

Some miss distances reflecting capture-radius requirements are shown in Figs. 2-6 for Aircraft A and B[5,6] as pursuer and evader, respectively. Aircraft B, a version of the F-4, is more maneuverable than A, a hypothetical Mach 3 design, at low and medium energies, in terms of sustainable turn rate, the sort of comparison appropriate for "dogfighting".[7] However, the region of advantage is smaller in a comparison of sustainable normal accelerations at equal speeds and altitudes (Figs. 7-9), and may be even smaller yet in a scenario allowing A greater choice of speed than B.

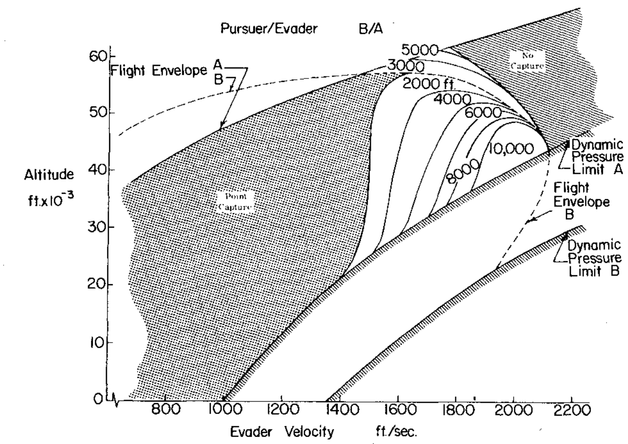

Fig. 5 **Capture radius required to counter evasive break – equal energies – noncoaltitude pursuit – pursuer/evader B/A.**

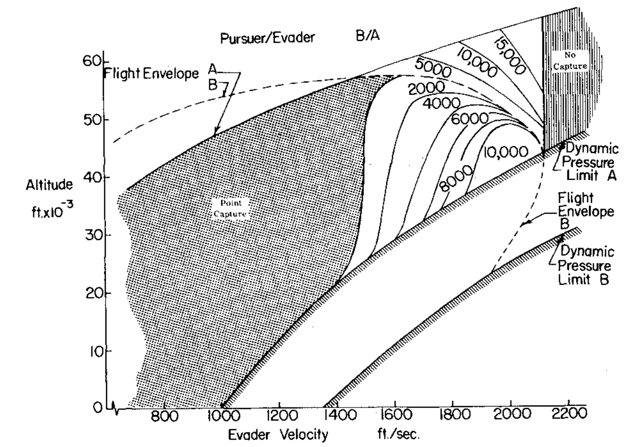

Fig. 6 **Capture radius required to counter evasive break – pursuer airspeed open – noncoaltitude pursuit – pursuer/evader B/A.**

Figure 2 presents capture-radius requirements for A pursuing B at various airspeeds and altitudes under the assumption that the pursuer's speed margin is zero in a *co-altitude* attack. The radius requirements of 1-5000 ft might be taken as realistic, while the larger figures, 10,000 ft and higher, are suspect in the light of the questionable matched-speed assumption in combination with the low airspeeds where they occur, i.e., there is a question whether it is reasonable to try filling such a maneuverability gap with weaponry. At two special points in the h-V plane, the required-capture-radius contours come together, implying that capture is possible for *any* capture-radius while it is impossible at nearby flight conditions! The two points are the intersections of the flight envelopes of the two aircraft, where, in fact, neither can maneuver, and the peculiar limiting behavior of the contours can be traced to the rigid co-altitude hypothesis.

Figure 3 presents results for the same marginal-overtaking-speed scenario as Fig. 2, but with the altitude restriction relaxed in favor of the pursuer, who is assumed to trade off altitude for airspeed prior to the breakaway (a "low-speed yo-yo" maneuver, in the jargon of the fighter pilot). The altitude sacrificed enters the miss computation root-sum-square fashion as a vertical component and is determined by a minimum-miss criterion, except at the lowest altitude (sea level in Fig. 3) where no tradeoff is permitted. The altitude differences work out to be of the order of hundreds of feet, ranging even to more than a thousand feet where large misses occur under a co-altitude assumption. It is seen that a large reduction of miss is obtained for the case of marginal-overtaking speed as a result of the pursuing aircraft's characteristic improvement in sustainable acceleration with speed right up to the placard limit. In the same spirit as the altitude-

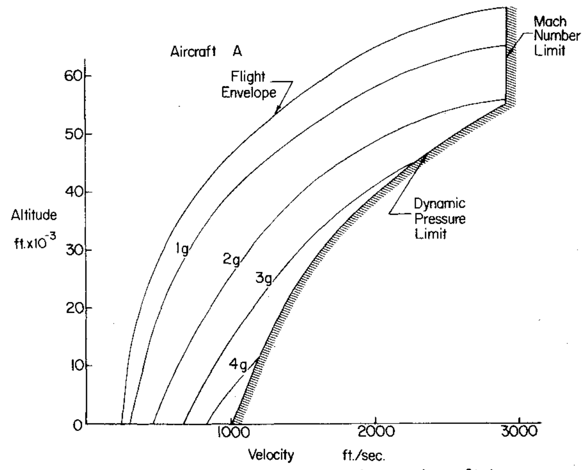

Fig. 7 **Sustainable acceleration – aircraft A.**

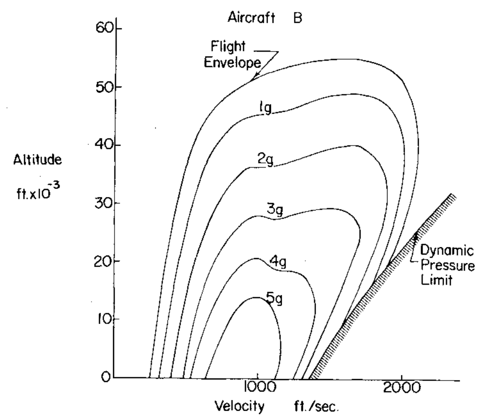

Fig. 8 **Sustainable acceleration – aircraft B.**

speed tradeoff freedom permitted the pursuer in the scenario of Fig. 3, points above the pursuer's flight envelope, but within the evader's, have been no longer scored automatically as "no capture," as in Fig. 2, but as misses in the thousands of feet, which include large vertical components.

Figure 4 presents capture-radius requirements with pursuer A's altitude matched to the evader B's, and pursuer airspeed chosen for maximum normal acceleration, which turns out to be the envelope airspeed for the particular aircraft. The largest capture-radius requirements are rather modest, about 1500 ft; however, it should be noted that the acceleration gap to be filled is not very big – about 1 g.

For A in the pursuer's role and B in the evader's, there is a difference in dynamic-pressure limits (1200 psf vs 2000 psf) that can be exploited by the evader: by fleeing at envelope speed at low altitude, he can require a capture-radius exceeding 15,000 ft, inasmuch as the pursuer must stay this far above him to keep within IAS placard. This related requirement shown in Figs. 2-4, exceeds the evasive-break capture-radius requirements; it indicates that gross deficiency in speed accountable to placard limits should be avoided in design against a threat, even with advanced weaponry.

Figure 5 presents results for the roles reversed, B pursuing A, under the assumptions of equal energies and pursuer freedom to trade off altitude for speed. Aircraft B has superior maneuverability and is capable of point capture up to speeds on the order of 1500 fps, above which there is a sharp increase in miss. At speeds above about 1800-2000 fps, B is outclassed in respect to performance and does not represent much of an offensive threat, even with large-radius weaponry. That part of the region above the pursuer's flight envelope, but within the evader's, which corresponds to higher specific energies than the pursuer can reach, has been scored as "no capture."

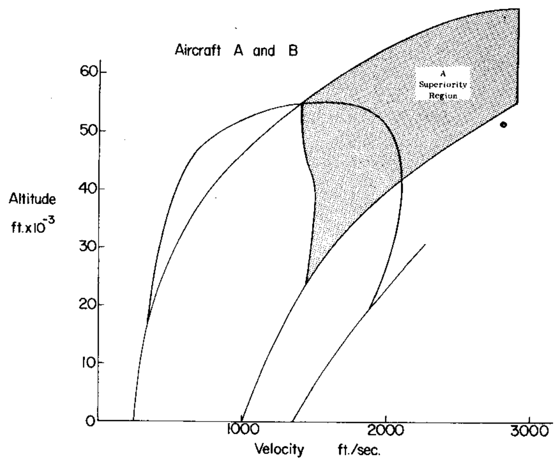

Fig. 9 Sustainable acceleration overlay – aircraft A and B.

Introducing freedom of speed choice for the pursuer has only a minor effect, as shown in Fig. 6, in that B's best speed choice is the minimum allowable (the evader's speed) in the critical part of the flight envelope. It should be noted also that the scenario that provides such freedom (a preliminary rectilinear chase) also provides similar freedom for A (barring surprise), who is better equipped to exploit it in the speed-altitude range of interest.

Concluding Remarks

The estimation formulas and procedures for studying weapon-radius requirements for attacks against maneuverable, but otherwise undefended, targets are intended for rough preliminary design study work. The computations presented are intended to convey a general impression of requirements and the way they vary with assumptions, although the example aircraft chosen are perhaps not particularly well matched. The results suggest that gaps in maneuverability can be filled to a considerable extent with large-radius weaponry, but that speed and ceiling gaps are less amenable. The constant-speed assumptions are restrictive, and results of the sort presented here mainly provide a frame of reference for additional computations examining the effects of storing energy and bleeding it off in the breakaway turns.

Features of the Homicidal- Chauffeur/Game-of-Two-Cars settings that make the rectilinear tail-chase/breakaway sequence central are unlimited fuel and visibility (complete information), and the evader's assumed complete lack of weaponry. These together lead to an unrealistic scarcity of forward-hemisphere passes in which large-radius weaponry tends to be effective. Some considerations on analyzing combat between vehicles both possessing weapons are given in Refs. 8 and 9.

Acknowledgment

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